DC3: A learning method for optimization with hard constraints

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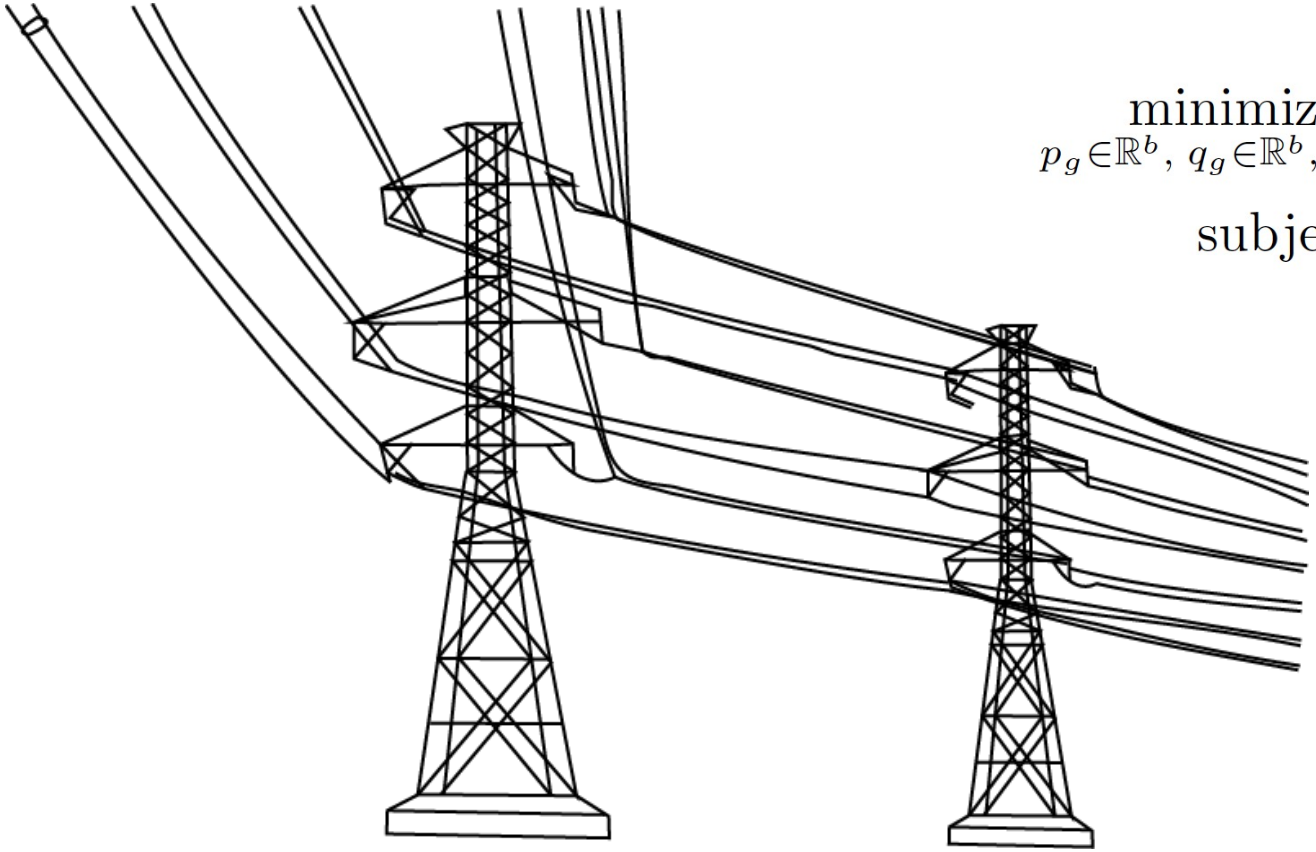
Motivation



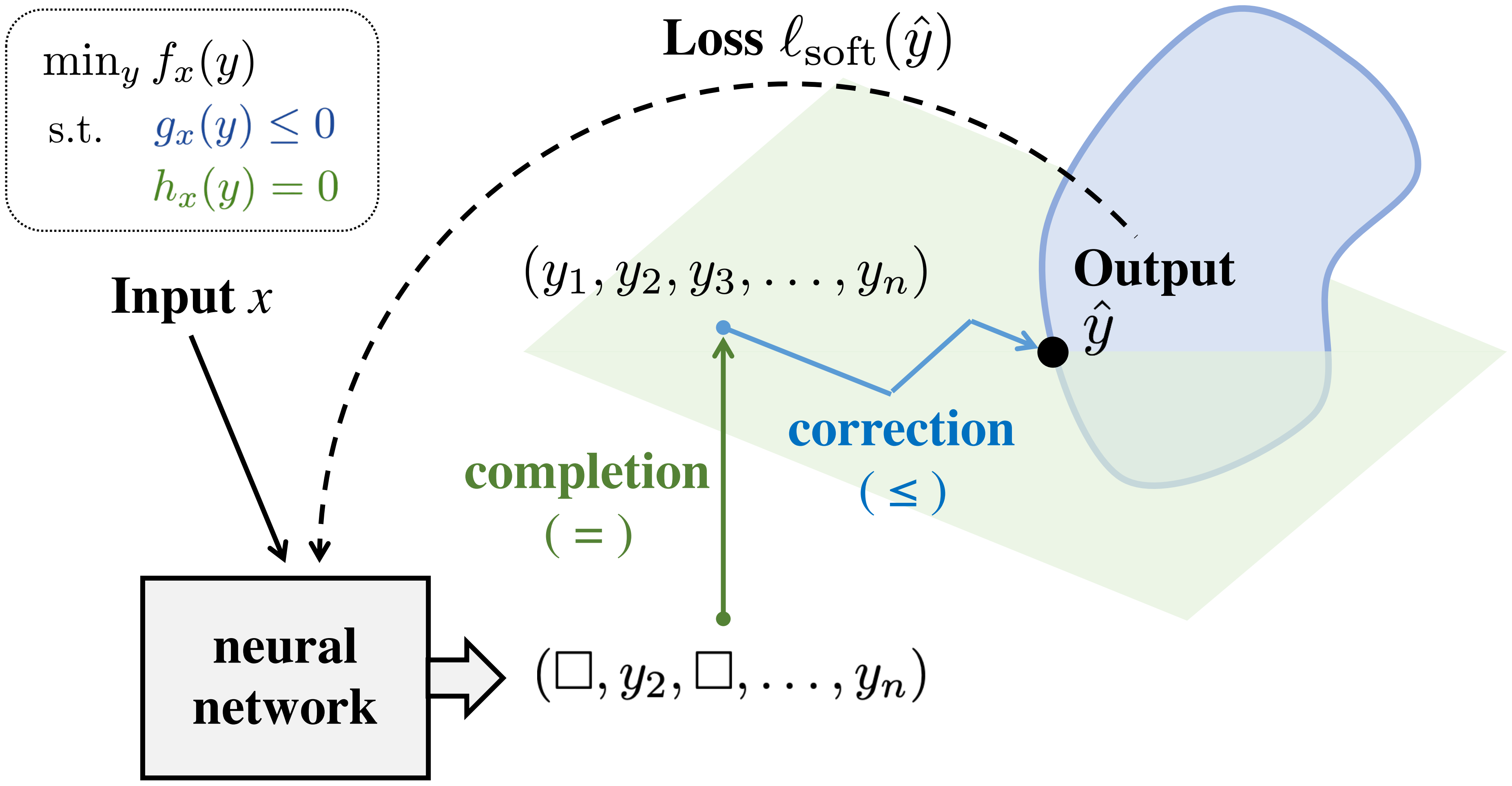
$$
\begin{aligned}
\underset{p_g\in\mathbb{R}^b,\ q_g\in\mathbb{R}^b,\ v\in\mathbb{C}^b}{\text{minimize}} \quad & p_g^T A p_g + b^T p_g \\
\text{subject to} \quad & p_g^{\min} \le p_g \le p_g^{\max} \\
& q_g^{\min} \le q_g \le q_g^{\max} \\
& v^{\min} \le |v| \le v^{\max} \\
& (p_g - p_d) + (q_g - q_d)i = \operatorname{diag}(v)\overline{W}\overline{v}.
\end{aligned}
$$

Problem setting

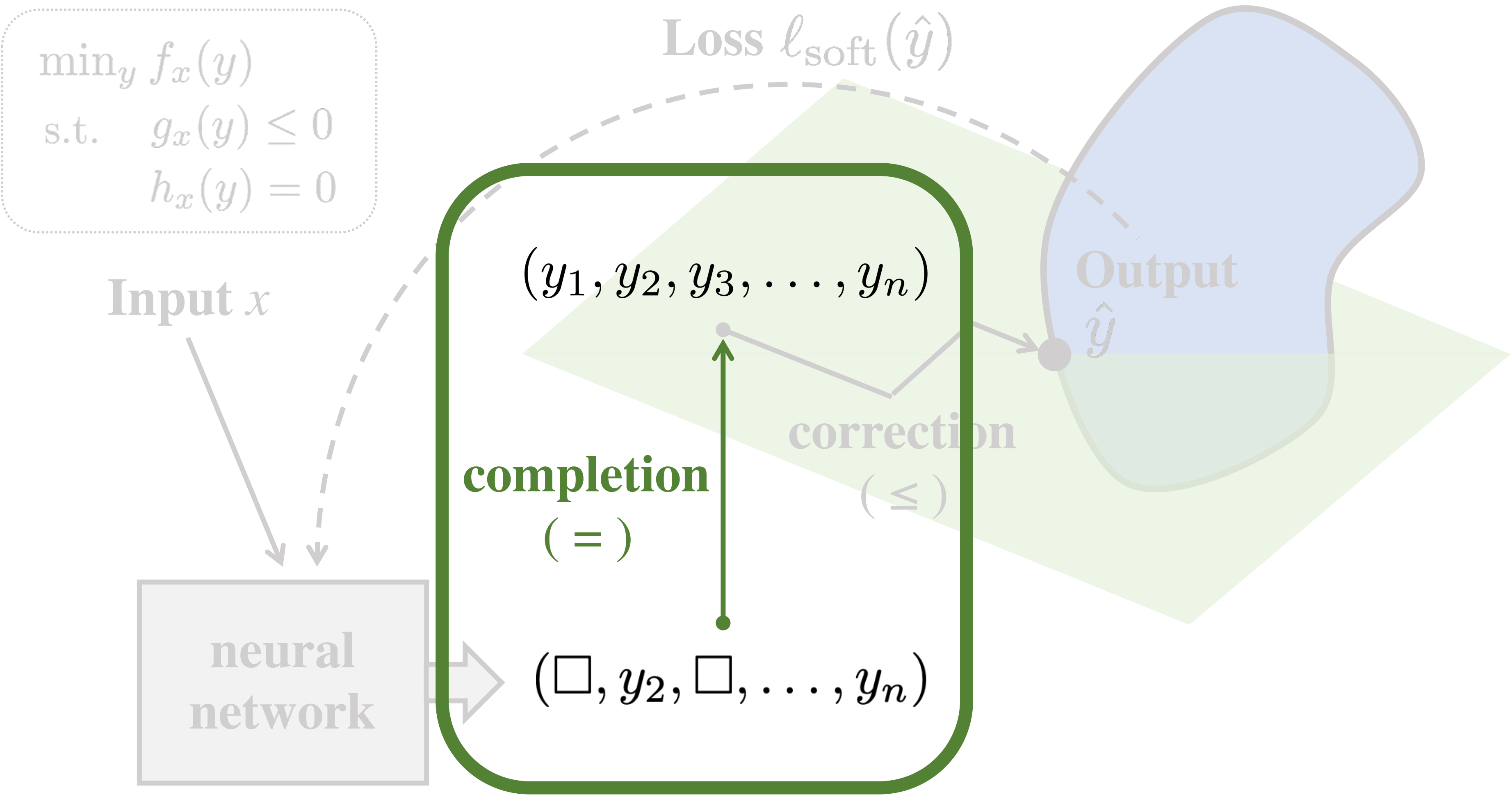
Goal: Approximate mapping from x to y, while satisfying constraints

$$
\begin{aligned}
\min_y \ & f_x(y) \\
\text{s.t.} \quad & g_x(y) \leq 0 \\
& h_x(y) = 0
\end{aligned}
$$

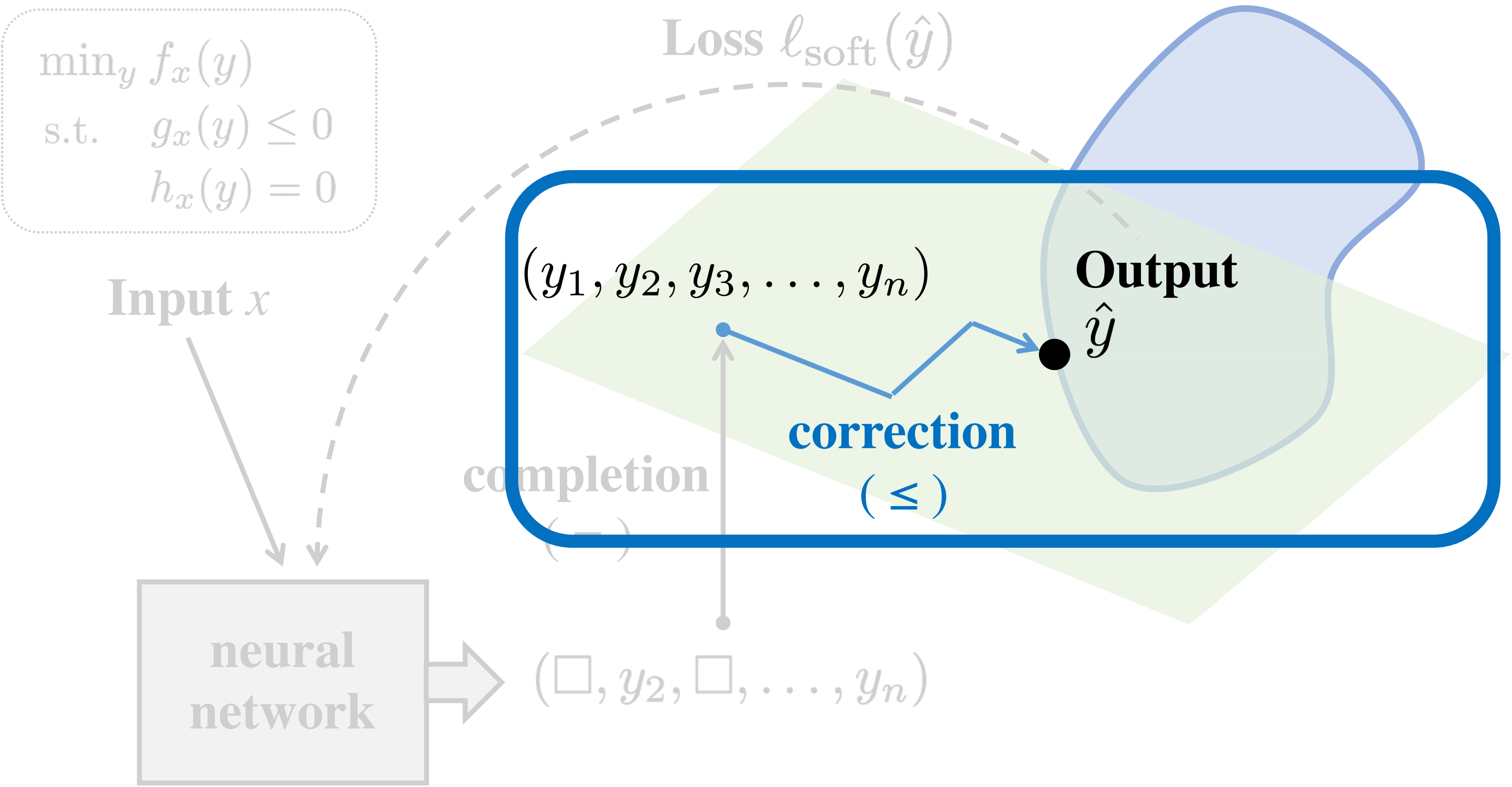
Overview of DC3



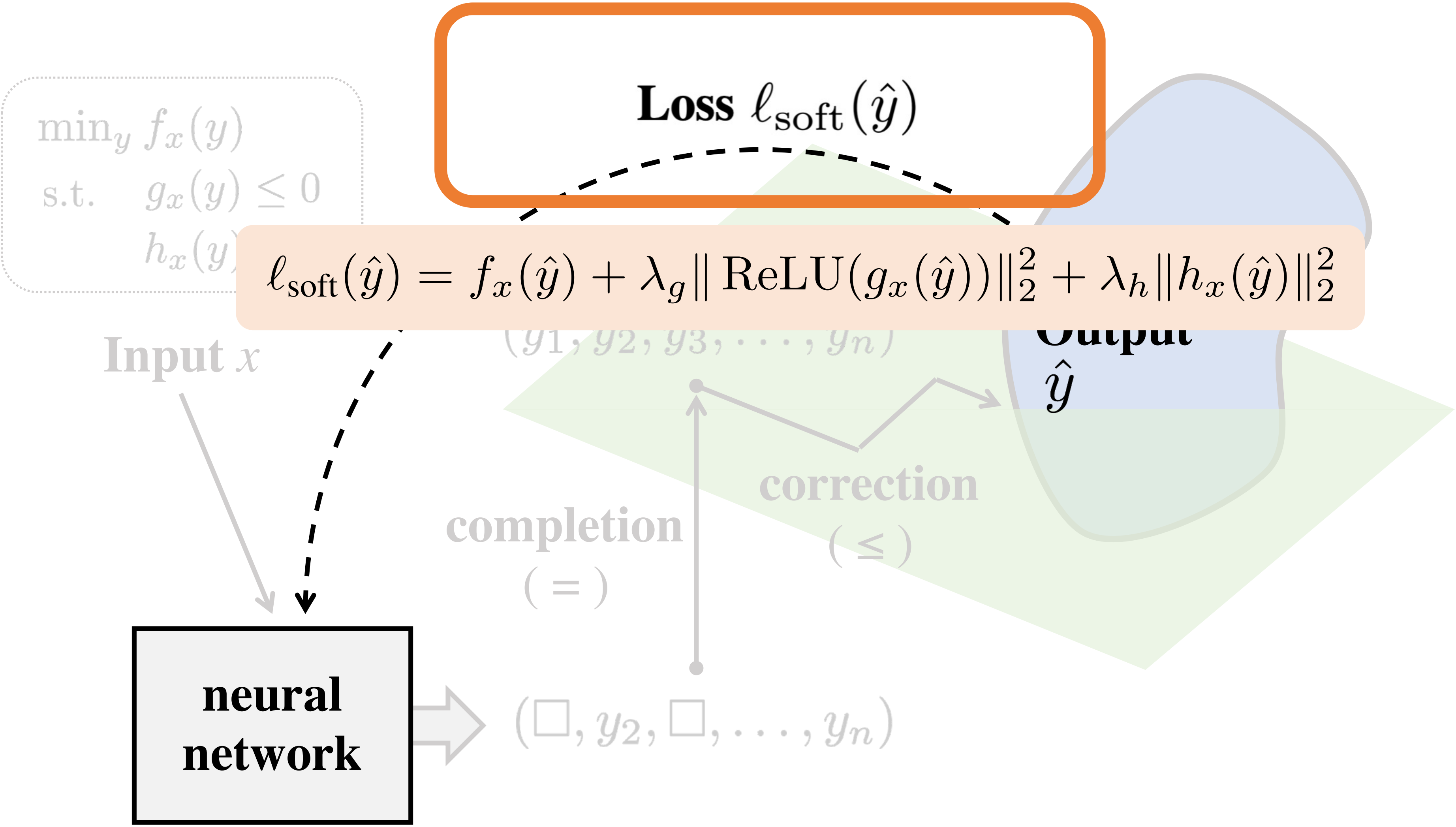
Overview of DC3



Overview of DC3

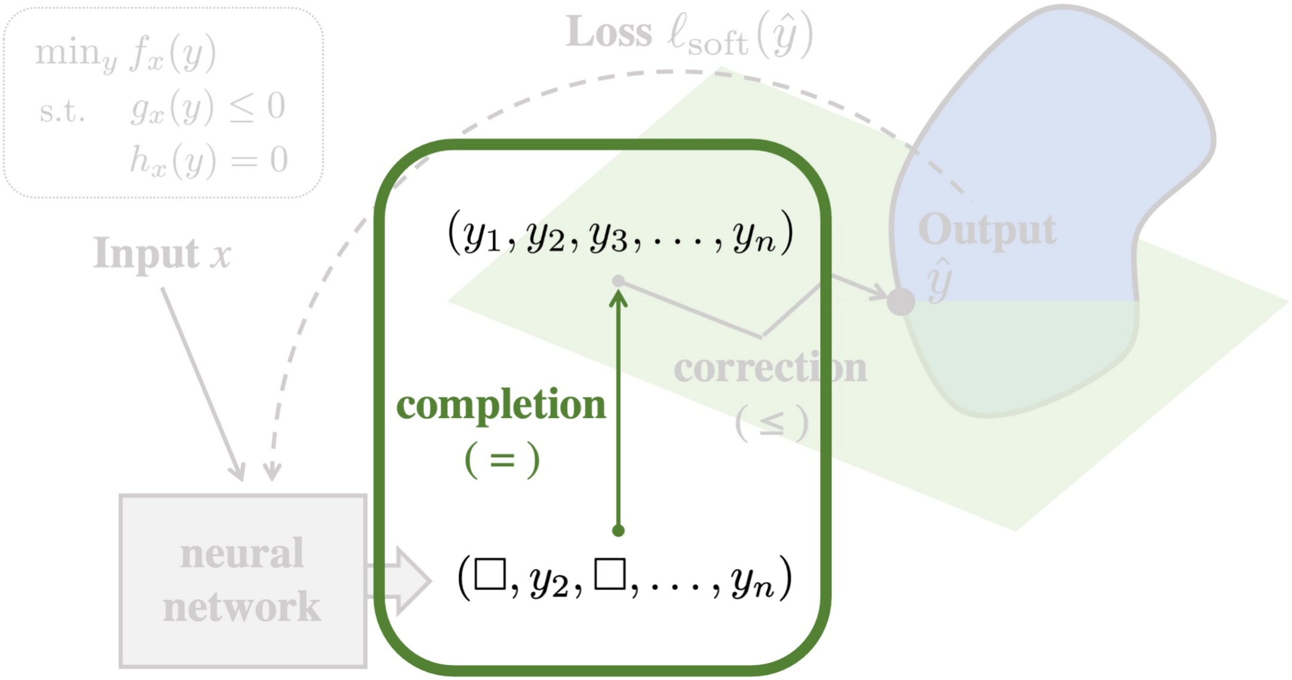


Overview of DC3



$$\ell_{\text{soft}}(\hat{y}) = f_x(\hat{y}) + \lambda_g \| \operatorname{ReLU}(g_x(\hat{y})) \|_2^2 + \lambda_h \| h_x(\hat{y}) \|_2^2$$

Equality completion



Output **subset of variables**

$$z = N_\theta(x)$$

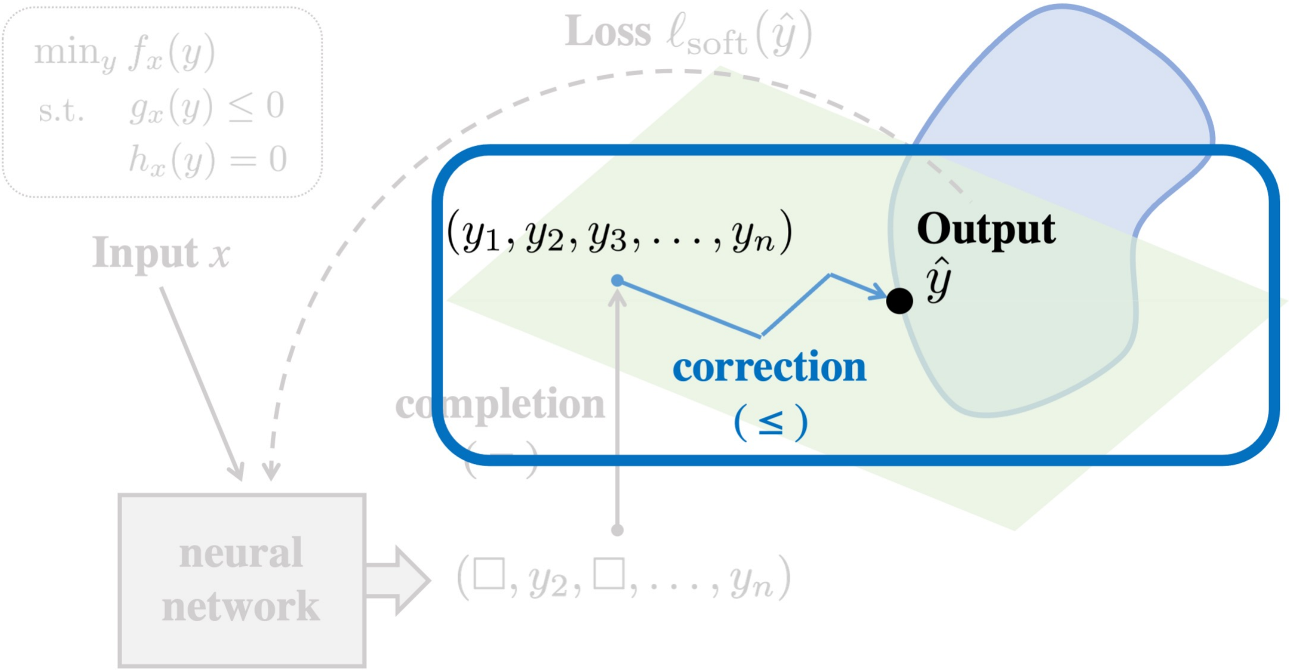
Then **solve for rest:** $\varphi_x(z)$

where $\varphi_x : \mathbb{R}^m \to \mathbb{R}^{n-m}$

s.t. $h_x([z^T \; \varphi_x(z)^T])^T = 0$

Procedure is **differentiable** (either explicitly or via implicit function thm)

Inequality correction



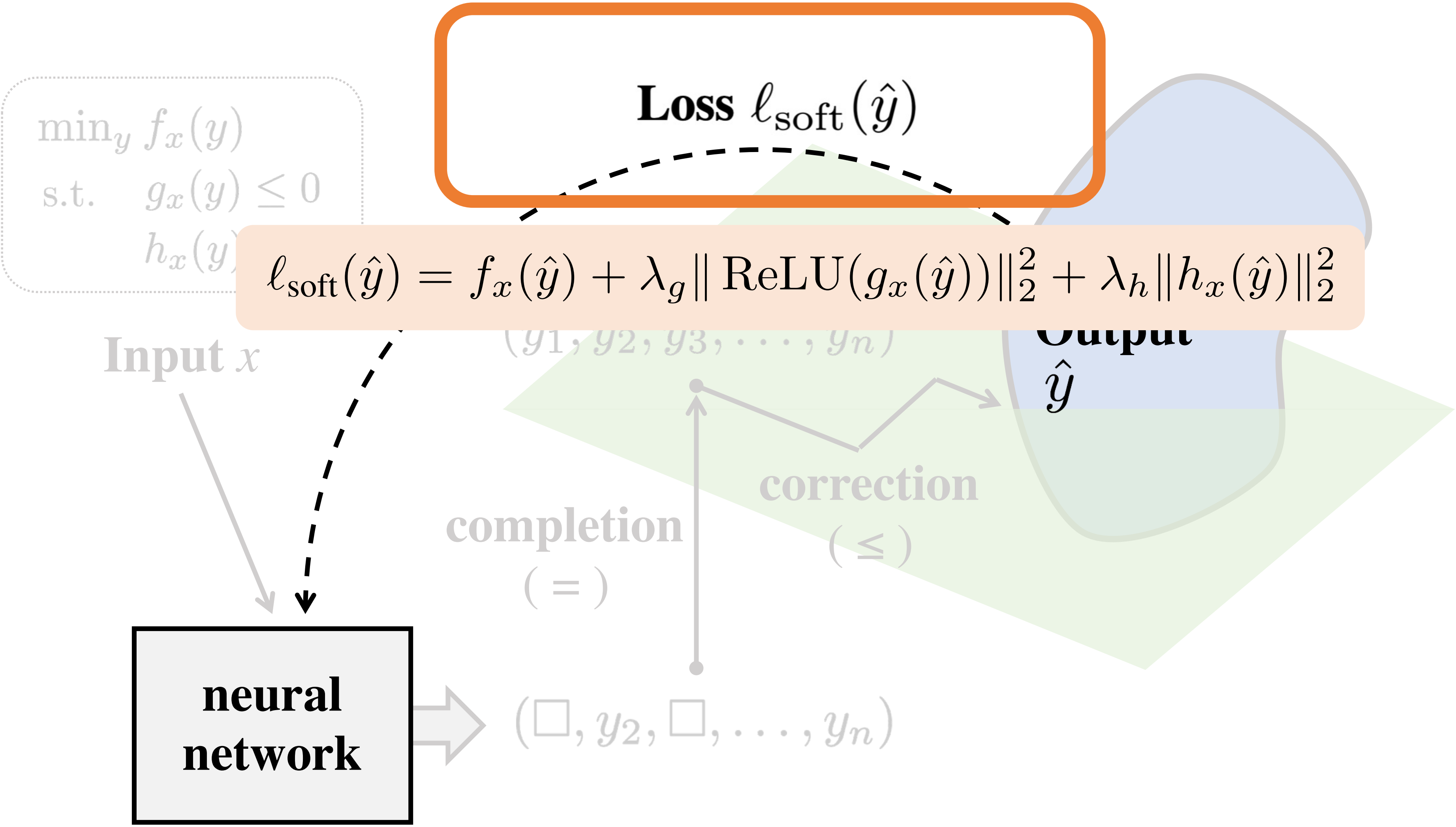
Gradient steps along manifold defined by equality constraints

$$\rho_x\left(\begin{bmatrix} z \\ \varphi_x(z) \end{bmatrix}\right) = \begin{bmatrix} z - \gamma\Delta z \\ \varphi_x(z) - \gamma\Delta\varphi_x(z) \end{bmatrix},$$

$$\text{for} \quad \Delta z = \nabla_z \left\| \mathrm{ReLU}\left(g_x\left(\begin{bmatrix} z \\ \varphi_x(z) \end{bmatrix}\right)\right)\right\|_2^2,$$

$$\Delta\varphi_x(z) = \frac{\partial \varphi_x(z)}{\partial z}\Delta z$$

End-to-end training with soft loss



$$\ell_{\text{soft}}(\hat{y}) = f_x(\hat{y}) + \lambda_g \|\operatorname{ReLU}(g_x(\hat{y}))\|_2^2 + \lambda_h \|h_x(\hat{y})\|_2^2$$

Experiments

Convex QP

$$\underset{y \in \mathbb{R}^n}{\text{minimize}} \; \frac{1}{2} y^T Q y + p^T y, \quad \text{s.t.} \; Ay = x, \; Gy \leq h$$

Simple non-convex

$$\underset{y \in \mathbb{R}^n}{\text{minimize}} \; \frac{1}{2} y^T Q y + p^T \sin(y), \quad \text{s.t.} \; Ay = x, \; Gy \leq h$$

AC optimal power flow

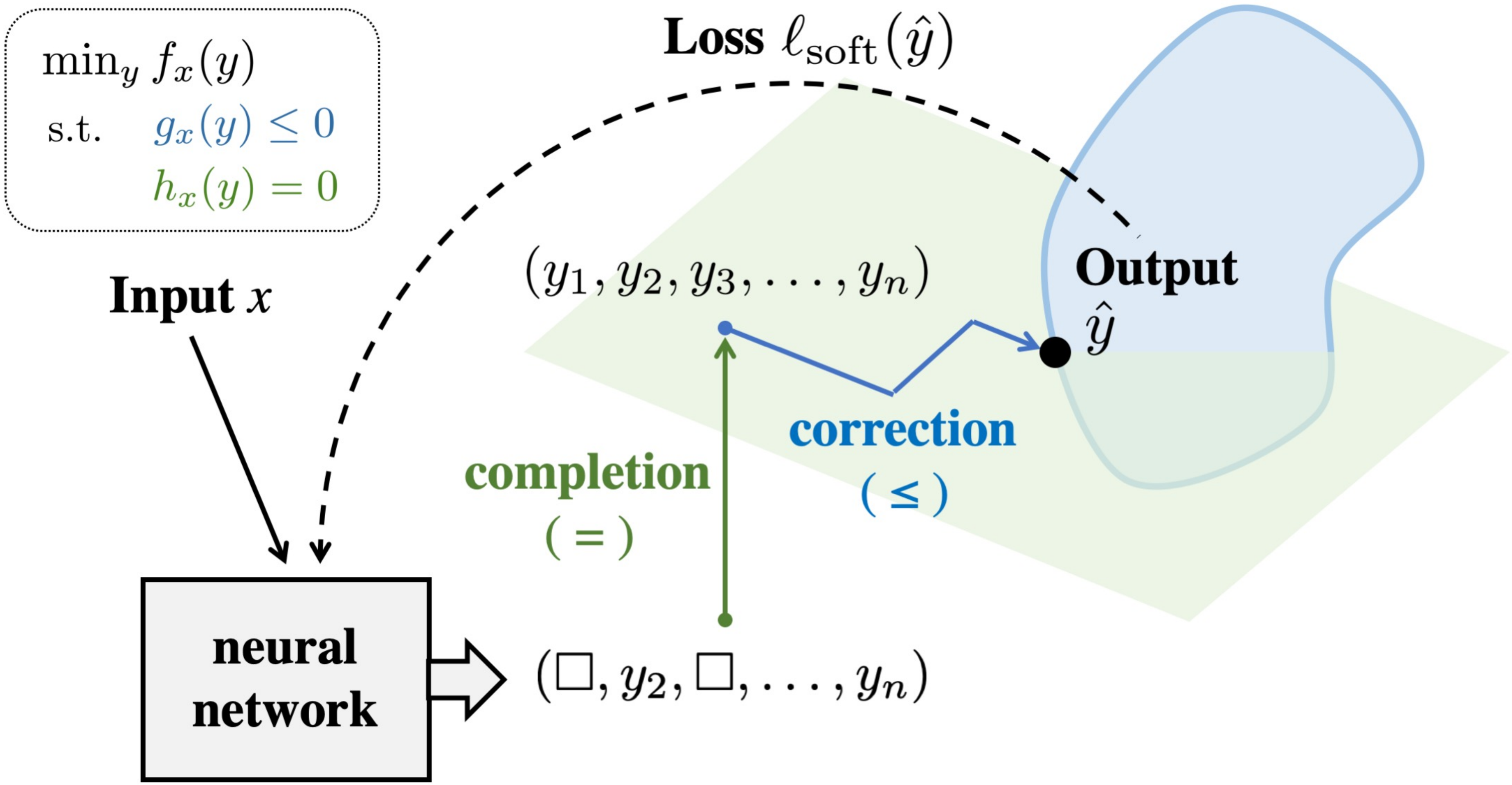
$$\begin{aligned} \underset{p_g \in \mathbb{R}^b, \, q_g \in \mathbb{R}^b, \, v \in \mathbb{C}^b}{\text{minimize}} \quad & p_g^T A p_g + b^T p_g \\ \text{subject to} \quad & p_g^{\min} \leq p_g \leq p_g^{\max} \\ & q_g^{\min} \leq q_g \leq q_g^{\max} \\ & v^{\min} \leq |v| \leq v^{\max} \\ & (p_g - p_d) + (q_g - q_d) i = \text{diag}(v) \overline{W} \overline{v} \end{aligned}$$

Results on AC optimal power flow

10x faster than optimizer, 0.22% optimality gap
Satisfies all constraints (unlike other DL methods)

	Obj. value	Max eq.	Mean eq.	Max ineq.	Mean ineq.	Time (s)
Optimizer	3.81 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.949 (0.002)
DC3	3.82 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.089 (0.000)
DC3, $\neq$	3.67 (0.01)	0.14 (0.01)	0.02 (0.00)	0.00 (0.00)	0.00 (0.00)	0.040 (0.000)
DC3, $\not\leq$ train	3.82 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.089 (0.000)
DC3, $\not\leq$ train/test	3.82 (0.00)	0.00 (0.00)	0.00 (0.00)	0.01 (0.00)	0.00 (0.00)	0.039 (0.000)
DC3, no soft loss	3.11 (0.05)	2.60 (0.35)	0.07 (0.00)	2.33 (0.33)	0.03 (0.01)	0.088 (0.000)
NN	3.69 (0.02)	0.19 (0.01)	0.03 (0.00)	0.00 (0.00)	0.00 (0.00)	0.001 (0.000)
NN, $\leq$ test	3.69 (0.02)	0.16 (0.00)	0.02 (0.00)	0.00 (0.00)	0.00 (0.00)	0.040 (0.000)
Eq. NN	3.81 (0.00)	0.00 (0.00)	0.00 (0.00)	0.15 (0.01)	0.00 (0.00)	0.039 (0.000)
Eq. NN, $\leq$ test	3.81 (0.00)	0.00 (0.00)	0.00 (0.00)	0.15 (0.01)	0.00 (0.00)	0.078 (0.000)

Summary



$\min_y f_x(y)$
s.t. $g_x(y) \leq 0$
$h_x(y) = 0$
Loss $\ell_{\text{soft}}(\hat{y})$
Input x
$(y_1, y_2, y_3, \ldots, y_n)$
Output
$\hat{y}$
completion
(=)
correction
(≤)
neural network
$(\square, y_2, \square, \ldots, y_n)$